The goal (introduction)

The goal of the MATLAB program is to model the flow of the concentration of a substance through a homogeneous porous media, obeying the advection-diffusion equation. This code is a direct translation to MATLAB off Bagus’ code, which he explains his motivation in his paper THIS PAPER. The program uses the Q1 Finite Element Method in order to solve the equation. This document shows the theory behind the code, explained as briefly and simply as possible for me at least. Keep in mind that this document is no means a complete guide to FEM, and should therefore be used as a crash course introduction to FEM in order to understand why the code is doing what it does. A great introduction to FEM is THIS, along with THIS as a short introduction to the lagrange multipliers used.

<http://www.softeng.rl.ac.uk/st/projects/felib3/Docs/html/Intro/intro-node68.html>

<http://arturo.imati.cnr.it/~marini/didattica/Metodi-engl/Intro2FEM.pdf>

The domain



Draw your own

State that in the code its using homogeneous shit

Explain the symbols

The Governing Equations in the domain

Advection Diffusion Equation

Where u = scalar field variable (eg concentration)

D = diffusion coefficient

**w** = velocity

f = source term

Boundary conditions

This condition means that u = 0 on the boundary between the perforations and the domain, shown in FIGURE X in red

This condition means that u = g on the boundary between the domain and the not domain. This is known as a Dirichlet boundary condition ( I think pls double check).

The Problem

The ADE is a second order PDE, meaning that the variable u is both globally continuous and smooth. This basically means that the derivative of u exists everywhere in the domain, and that there are no ‘jumps’ in the graph.

PUT IN FIGURES SHOWING SMOOTHNESS AND CONTINUOUSNESS.

Numerical simulations return numerical values at discreet points in the domain, which means that you can have the function globally continuous, but it definitely cannot be globally smooth.

FIGURE SHOWING WHAT IT MEANS

The ADE in the second order PDE form is written in what is known as the strong formulation with strong solutions. We would need to rewrite the problem using the weak formulation, which mean that the solution is not required to have an existing derivative everywhere.

* The ADE is a second order PDE, which means that u is both continuous and very smooth
* Numerical simulations work with numbers instead of functions, meaning that the solution a computer outputs is globally continuous but not globally differentiable.
* The ADE in the second order PDE is written in what is known as the ‘strong form’, so we need to convert it to a weak form

The heart of converting the strong form to the weak form is Green’s Formula, shown below.

Substituting the Laplacian from our stuff, we get our weak formulation.

Greens theorem



Sub in

Here, v is a test function, EXPLAIN WHAT A TEST FUNCTION IS.

Discretizing the domain.

Something something approximating u as a series of linear equations